20 $Sp(2N)$ and Chiral Theories

20.1 Duality for $Sp(2N)$

	$Sp(2N)$	$SU(2F)$	$U(1)_R$
Q	$\Box$	$\Box$	$\frac{F-1-N}{F}$

We'll use the notation that $Sp(2) \sim SU(2)$. Recall that the adjoint of $Sp(2N)$ is the two-index symmetric tensor.

Irrep	$d(r)$	$T(r)$
$\yng(1)$	$2N$	1
$\yng(1,1)$	$N(2N-1)-1$	$2N-2$
$\yng(2)$	$N(2N+1)$	$2N+2$
$\yng(1,1,1)$	$\frac{N(2N-1)(2N-2)}{3} - 2N$	$\frac{(2N-3)(2N-2)}{2} - 1$
$\yng(3)$	$\frac{N(2N+1)(2N+2)}{3}$	$\frac{(2N+2)(2N+3)}{2}$
$\yng(2,1)$	$\frac{2N(2N-1)(2N+1)}{3} - 2N$	$(2N)^2 - 4$

The one-loop β function coefficient for $N > 4$ is

$$b = 3(2N+2) - 2F \tag{20.1}$$

The moduli space is parameterized by a "meson"

$$M_{ji} \quad = \quad \Phi_j \Phi_i \tag{20.2}$$

The flavor singlet chiral superfields transform as:

$$\begin{array}{ccc} & U(1)_A & U(1)_R \\ \Lambda^{b/2} & 2F & 0 \\ \mathrm{Pf} M & 2F & 2(F-1-N) \end{array} \tag{20.3}$$

Where

$$\mathrm{Pf} M = \epsilon^{i_1 \ldots i_{2F}} M_{i_1 i_2} \ldots M_{i_{2F-1} i_{2F}} \ . \tag{20.4}$$

So we see it is possible to generate a dynamical superpotential

$$W_{\mathrm{dyn}} \propto \left(\frac{\Lambda^{\frac{b}{2}}}{\mathrm{Pf} M} \right)^{\frac{1}{N+1-F}} \tag{20.5}$$

for $F < N+1$. For $F = N+1$ one finds confinement with chiral symmetry breaking

$$\mathrm{Pf} M = \Lambda^{2(N+1)} \ . \tag{20.6}$$

For $F = N+2$ one finds confinement without chiral symmetry breaking with a superpotential:

$$W = \mathrm{Pf} M \tag{20.7}$$

A solution to the anomaly matching for $F > N-2$ is given by:

	$Sp(2(F-N-2))$	$SU(2F)$	$U(1)_R$
q	$\Box$	$\overline{\Box}$	$\frac{N+1}{F}$
M	$\mathbf{1}$	$\Box\Box$	$\frac{2(F+2-N)}{F}$

This theory admits a unique superpotential:

$$W = \frac{M_{ji}}{\mu} \phi^j \phi^i \tag{20.8}$$

For $3(N+1)/2 < F < 3(N+1)$ we have an infrared fixed point. For $N+3 \leq F \leq 3(N+1)/2$ the dual is infrared free.

20.2 Why Chiral Gauge Theories are Interesting/Confusing

We would like to use our new non-perturbative methods for understanding SUSY gauge theories to analyse dynamical SUSY breaking. Usually vector-like gauge theories don't break SUSY, while chiral gauge theories can. If a theory is vector-like we can give masses to all the matter fields. If these masses are large we have a pure gauge theory that has gaugino condensation but no SUSY breaking. By Witten's index argument we can vary the mass but the number of bosonic minus fermionic vacua doesn't change. If taking the mass to zero does move some vacua out to infinity, then the massless theory has the same number of vacua, and SUSY is not broken.

As our first example of a chiral gauge theory consider

	$SU(N)$	$SU(N+4)$
$\overline{Q}$	$\overline{\Box}$	$\Box$
T	$\Box\Box$	$\mathbf{1}$

we will not write down the charges under the two $U(1)$'s. This theory is dual to

	$SO(8)$	$SU(N+4)$
q	$\square$	$\overline{\square}$
p	$\mathbf{S}$	$\mathbf{1}$
$U \sim \mathrm{det}T$	$\mathbf{1}$	$\mathbf{1}$
$M \sim \overline{Q}T\overline{Q}$	$\mathbf{1}$	$\square\square$

with a superpotential

$$W = Mqq + Upp \tag{20.9}$$

This theory is vector-like! The dual β function coefficient is:

$$b = 3(8-2) - (N+4) - 1 = 13 - N \tag{20.10}$$

So the dual is IR free for $N > 13$.

20.3 S-Confinement

We need some semi-systematic way to survey chiral gauge theories. One way to do this is to generalize well understood dual descriptions. The simplest of these is confinement without chiral symmetry breaking in $SU(N)$ with $N+1$ flavors. Recall the confined description had a superpotential

$$W = \frac{1}{\Lambda^{2N-1}} \left(\mathrm{det}M - BM\overline{B} \right) \tag{20.11}$$

The crucial features of this description were that since there was no chiral symmetry breaking and that the meson-baryon description was valid over the whole moduli space. That is there was a smooth description with no phase-transitions. This is because the theory obeyed complimentarity, every static source could be screened by the squarks. To generalize this we will need to have fields that are fundamentals of SU or Sp and spinors of SO. We will only consider theories that have a superpotential in the confined description. This requirement gives us and index constraint. Theories that satisfy these conditions are called s-confining.

Consider a gauge theory with one gauge group and arbitary matter fields. Choose an anomaly free $U(1)_R$ such that ϕ_i has charge q while all other fields have zero charge. The charge q is determined by anomaly cancelation:

$$0 = qT(r_i) + T(Ad) - \sum_j T(r_j) \tag{20.12}$$

Since we can do this for any field, and for each choice the superpotential has R charge 2 we have

$$W \propto \Lambda^3 \left[\Pi_i \left(\frac{\phi_i}{\Lambda} \right)^{T(r_i)} \right]^{2/(\sum_j T(r_j) - T(Ad))} \tag{20.13}$$

There may in general be a sum of terms corresponding to different contractions of gauge indices. Requiring that this superpotential be homorphic at the origin means there should be integer powers of the composite fields, which implies integer powers of the fundamental fields. Unless all the $T(r_i)$ have a common divisor we must have

$$\begin{aligned} \sum_j T(r_j) - T(Ad) &= 1 \text{ or } 2 \text{ for } SO \text{ or } Sp \\ 2(\sum_j T(r_j) - T(Ad)) &= 1 \text{ or } 2 \text{ for } SU \end{aligned} \tag{20.14}$$

The differing cases come from the different conventions for normalizing generators, for SO and Sp we have $T(\Box) = 1$, while for SU we have $T(\Box) = 1/2$. Anomaly cancelation for SU and Sp require that the left hand side be even. This condition is necessary for s-confinement, but not sufficient. One has to check explicitly that for SO the sum has to be 1. Thus we have

$$\sum_j T(r_j) - T(Ad) = \begin{cases} 1 \text{ for } SU \text{ or } SO \\ 2 \text{ for } Sp \end{cases} \tag{20.15}$$

This condition gives a finite list of candidate s-confining theories.

We can check whether candidate theories that satisfy the index constraint really are s-confining by going out in moduli space. Generically we break to theories with smaller gauge groups and singlet fields that decouple in the infrared. If the smaller gauge theory is not s-confining the the original theory was not s-confining. Alternatively if we have an s-confining theory and we go out in moduli space we must end up with another s-confining theory. Using these checks one can go through the list of candidates. For SU one finds that the following theories are the only ones that satisfy the conditions for s-confinement.

$SU(N)$	$(N+1)(\square+\overline{\square})$
$SU(N)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix} + N\overline{\square} + 4\square$
$SU(N)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix} + \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + 3(\square+\overline{\square})$
$SU(5)$	$3(\begin{smallmatrix}\square\\\square\end{smallmatrix} + \overline{\square})$
$SU(5)$	$2\begin{smallmatrix}\square\\\square\end{smallmatrix} + 2\square + 4\overline{\square}$
$SU(6)$	$2\begin{smallmatrix}\square\\\square\end{smallmatrix} + 5\overline{\square} + \square$
$SU(6)$	$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} + 4(\square+\overline{\square})$
$SU(7)$	$2(\begin{smallmatrix}\square\\\square\end{smallmatrix} + 3\overline{\square})$

Lets consider the special case:

	$SU(2N+1)$	$SU(4)$	$SU(2N+1)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	1	1	0	$2N+5$	0
$\overline{Q}$	$\overline{\square}$	1	$\square$	4	$-2N+1$	0
Q	$\square$	$\square$	1	$-2N-1$	$-2N+1$	$\frac{1}{2}$

This theory has a confined description

	$SU(4)$	$SU(2N+1)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$Q\overline{Q}$	$\square$	$\square$	$3-2N$	$-4N+2$	$\frac{1}{2}$
$A\overline{Q}^2$	1	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	8	$-2N+7$	0
A^NQ	$\square$	1	$-2N-1$	$2N^2+3N+1$	$\frac{1}{2}$
$A^{N-1}Q^3$	$\overline{\square}$	1	$-6N-3$	$2N^2-3N-2$	$\frac{3}{2}$
$\overline{Q}^{2N+1}$	1	1	$4(2N+1)$	$-4N^2+1$	0

with a superpotential

$$W = \frac{1}{\Lambda^{2N}}\Big[(A^NQ)(Q\overline{Q})^3(A\overline{Q}^2)^{N-1} + (A^{N-1}Q^3)(Q\overline{Q})(A\overline{Q}^2)^N + (\overline{Q}^{2N+1})(A^NQ)(A^{N-1}Q^3)\Big] \tag{20.16}$$

The equations of motion reproduce the classical constraints, and integrating out a flavor gives confinement with chiral symmetry breaking.

20.4 Deconfinement

Consider the odd N theory with $F \geq 5$

	$SU(N)$	$SU(F)$	$SU(N+F-4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{1}$	$\mathbf{1}$	0	$-2F$	$\frac{-12}{N}$
Q	$\square$	$\square$	$\mathbf{1}$	1	$N-F$	$2-\frac{6}{N}$
$\overline{Q}$	$\overline{\square}$	$\mathbf{1}$	$\square$	$\frac{-F}{N+F-4}$	F	$\frac{6}{N}$

We can imagine that A is a composite meson of a s-confining Sp theory

	$SU(N)$	$Sp(N-3)$	$SU(F)$	$SU(N+F-4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
Y	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$	0	$-F$	$\frac{-6}{N}$
Z	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	0	FN	8
$\overline{P}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	0	$F(1-N)$	$6-\frac{6}{N}$
Q	$\square$	$\mathbf{1}$	$\square$	$\mathbf{1}$	1	$N-F$	$2-\frac{6}{N}$
$\overline{Q}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\frac{-F}{N+F-4}$	F	$\frac{6}{N}$

with a superpotential

$$W = YZ\overline{P} \tag{20.17}$$

The $SU(N)$ group has $N+F-3$ flavors so we can using our standard duality: to find another dual:

	$SU(F-3)$	$Sp(N-3)$	$SU(F)$	$SU(N+F-4)$
y	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$
$\overline{p}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
q	$\square$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$
$\overline{q}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$
M	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$
L	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\square$
B	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$

with

$$W = Mq\overline{q} + Bq\overline{p} + Ly\overline{q} \tag{20.18}$$

But $Sp(N-3)$ with $N+2F-7$ fundamentals has an $Sp(2F-8)$ dual:

	$SU(F-3)$	$Sp(2F-8)$	$SU(F)$	$SU(N+F-4)$
$\widetilde{y}$	$\overline{\square}$	$\square$	$\mathbf{1}$	$\mathbf{1}$
$\overline{p}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
q	$\square$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$
$\overline{q}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$
M	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$
l	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\overline{\square}$
B	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$
a	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
H	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$
(Ly)	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\square$

with

$$W = a\widetilde{y}\widetilde{y} + Hll + (Ly)l\widetilde{y} + Mq\overline{q} + Bq\overline{p} + (Ly)\overline{q} \tag{20.19}$$

which, after integrating out (Ly) and $\overline{q}$ becomes

$$W = a\widetilde{y}\widetilde{y} + Hll + Mql\widetilde{y} + Bq\overline{p} \tag{20.20}$$

With $F = 5$ we have a gauge group $SU(2) \times SU(2)$ and one can show (using the fact that gauge invariant operators have dimensions larger than one) that for $N > 11$ this theory has an IRFP. One can also show that some of the fields are IR free. Integrating out one flavor completely breaks the gauge group and the light degrees of freedom are just the composites of the s-confining description. With the other duals we would have to discuss strong interaction effects to see that we get the correct confined description.

References

[1] "Lectures on supersymmetric gauge theories and electric-magnetic duality," by K. Intriligator and N. Seiberg, hep-th/9509066.

[2] C. Csaki, M. Schmaltz and W. Skiba, "Systematic approach to confinement in N = 1 supersymmetric gauge theories," Phys. Rev. Lett. **78** (1997) 799 hep-th/9610139.

[3] C. Csaki, M. Schmaltz and W. Skiba, "Confinement in N = 1 SUSY gauge theories and model building tools," Phys. Rev. **D55** (1997) 7840 hep-th/9612207.

[4] P. Pouliot and M.J. Strassler, "A Chiral $SU(N)$ Gauge Theory and its Non-Chiral $Spin(8)$ Dual," Phys. Lett. **B370** (1996) 76 hep-th/9510228.

[5] P. Pouliot, "Duality in SUSY $SU(N)$ with an Antisymmetric Tensor," Phys. Lett. **B367** (1996) 151 hep-th/9510148.

[6] J. Terning, "Duals for SU(N) SUSY gauge theories with an antisymmetric tensor: Five easy flavors," Phys. Lett. **B422** (1998) 149 hep-th/9712167.